

MARKOV MODELS FOR ASSESSMENTS AND PREDICTION OF STRUCTURAL ELEMENTS

Abstract

Purpose. The purpose of the work is to develop a universal model for assessing the technical condition of the structure in the function of the time of operation. The theoretical basis for the development of a stochastic model for the technical condition of a structure during operation is the Markov theory of random processes. Phenomenological models of cumulative accumulation of damaged due to the natural degradation of elements during the life cycle of operation are considered. The wear of the structure element is described by a Markov discrete process with continuous time. The Markov process, whose evolution over time depends only on a fixed modern state, has found wide application in the system of operation of road bridges. Degradation of elements during operation is considered as a stream of failures that are physically a manifestation of damage to the elements of a structure under the influence of loads and the environment. The degradation of bridge elements is interpreted as a stationary simplest flow of Poissian type. A mathematical model of a random process with continuous time and discrete states is described by the well-known Kolmogorov – Chapman equations.

The technique. Theoretical study of the processes of degradation of elements of bridges made in the framework of probability theory and mathematical statistics.

Results. The model for assessment and prediction of a technical state of structural elements which is based on the Markov theory of random processes is received.. Markov stochastic models of damage accumulation as a result of natural deterioration, considered herein, are universal. They are well proved theoretically and have a practical orientation. These models may be applied as an effective tool for technical state assessments and prediction of structural residual resource. A key aspect of this approach is the special procedure for evaluating the transition intensity parameter. The main lack of the model is that the basis of the elementary flow with parameter $\lambda = \text{const}$ is accepted. However, the parameter λ is determined for each given element, and every time after a subsequent inspection it can be specified, so the accuracy of the model should increase.

Scientific novelty. The study is pioneering. For the first time in the system of operation of structural elements, a stochastic assessment of the technical condition of the structure is proposed.

Practical value. The resulting model is a practical tool for managing the reliability and resource of structural elements.

Key words: deterioration process; intensity of failures; Markov model of a random process; matrix of transitions; reliability function, structural elements.

Introduction

This research dedicated the problem of stochastic modelling to describe damages accumulated in structural elements. Classic scientific a priori formulation of structure lifecycle will obtain here a rigorous scientific substantiation in resource terms – structures find corresponding universal models describing phenomenological processes of destruction with application of Markovian random functions. The basic scientific idea of this approach lies in *new paradigm of structural theory – connection between limit state equations and service life duration*, so-called ‘stochastic’ ‘time-dependent’

random variables problem and the related failure probability calculations. The problem has its own weight as a risk management tool.

There have been multiple attempts to develop analytical models representing the degradation of bridge structures by knowing the deterioration process of materials. These models are typically very complex, as they attempt to represent physical and chemical processes using specific models for each different degradation cause. Moreover, they have to account that the actual condition of a bridge is the consequence of many concurring degradation processes. For these reasons, obtaining an effective and reliable mathematical degradation model is a very difficult task.

A different approach allows for evaluating the degradation a reliability-based models, using historical bridge conditional al data. This allows an appropriate degradation curve to be determined for each bridge reflecting all specific factors that have caused its degradation.

Despite classic, basic research in the reliability theory and practical algorithms of estimation and prediction of structural condition states are insufficiently developed [1, 2, 3, 8, 12, 13, 14].

The classic models of reliability are based on the basic mechanics of material destruction, consequently the structural reliability theory has not had wide practical application. The main limitation is that the classic reliability models require precise information based on a load history, which is expressed in the number of loading – unloading cycles [2]. Such data, for the majority of structural elements, have never been obtained and it is not expected to be done so in the foreseeable future. Therefore, when modelling uncertain processes of structural deterioration, one needs a model focused on incompleteness of the initial information. This limitation can be overcome by a phenomenological stochastic model.

The phenomenological stochastic model, which is described as the accumulation of damages in the process where the evolution of time is defined by probabilistic laws, has become a powerful alternative for a deterministic model during the last 30-40 years. Now, many researchers are inclined to believe that the *Markov stochastic model* are the integrated universal for describe the cumulative damage of structural elements.

The objective of this paper is to present this universal approach for evaluation stochastic deterioration of the structural elements under maintenance.

Markovian models of random processes

Theoretical basis of the research is Markovian mathematical model for estimation and prediction of structure element service life prediction within its operational cycle. This model is one whose evolution in time depends only of fixed current state.

The modelling by the Markov process means that for any time t_0 the probability of stay in an element, in each state in the future ($t > t_0$) depends only upon its state in the present ($t = t_0$). It is a so-called memory-less process with discrete states ($1, 2 \dots n$) in continuous time t . It is a «memory-less» process because each discrete state is independent of the previous state. In other words, the distribution of the Markov process at the moment of time, t , can be expressed through the distribution of the previous moment of time independently of the process history.

At the same time the Markov discrete process is not completely independent from the past. The small amount of the philosophical formulation of this dependence is: for the Markov discrete process a future and its correlation depend on the past only through the present. Hence, it is not complete independence of a future from the past, because the starting, present state ($t = t_0$) depends from the past deterioration stochastic process.

Here we shall formulate discrete models which meander on discrete states that transit in one direction only - from a state with a smaller number to a state with the greatest number. In terms of the Markov process, the task is to find unconditional probabilities for stay system **S** on any step k in a state S_i :

$$p_i(k) = \text{Prob}[S(k) = S_i]; k = 1, 2, \dots, n; i = 0, 1, \dots, n - 1 \quad (1)$$

The probabilities $p_i(k)$ are expressed through conditional probabilities of transition of system **S** on a step k in a state S_j , provided that on a step $k-1$ the system is able to S_i :

$$p_{ij}(k) = \text{Prob}[S(k) = S_j \mid S(k-1) = S_i] \quad i, j = 0, 1, \dots, n-1 \quad (2)$$

It is apparent that the transition probabilities (2) form a square matrix of transitions with the size $n \cdot n$, where n is the number of discrete states considered. Let us denote the transitions matrix as $\mathbf{P}(i, t)$. On the main diagonal of the matrix, $\mathbf{P}(i, t)$ stands for the probabilities of a system delay in that given state, where S_i is on a step k ; on lateral diagonals, a transition probability of the system transition is from the state S_i to the state S_j - $p_{ij}(k)$.

In the search of transition probabilities, which are contained in the stochastic matrix $\mathbf{P}(i, t)$, a major central point in the development of a deterioration model for structure elements are described by the Markov discrete process [5, 6]. When the matrix, $\mathbf{P}(i, t)$, of a conditional transition probability is found, the unconditional probabilities P_j of that system stay in a state S_i are turned out by the recurrent formula:

$$P_t(k) = \sum_{j=1}^n P_j(k-1)P_{ji}, \quad k = 1, 2, \dots, n, \quad i = 1, 2, \dots, n, \quad (3)$$

where i is number of current state; n is number of discrete states in the course of element lifecycle; p_k is *absolute probability* of element remaining in k -th discrete state; $p_{ik}(t)$ is transient probability of k -th discrete state.

Required probabilities of the Markov chain $p_1(t), p_2(t), \dots, p_n(t)$ are functions of time. They are determined from a system of ordinary differential equations. It is known as the forward Chapman - Kolmogorov equation, which describes the evolution of the Markov discrete process with continuous time. In the matrix form, the differential equations are:

$$\frac{d\mathbf{P}(i, t)}{dt} = \mathbf{P}(i, t) \cdot \mathbf{E}, \quad (4)$$

where $\mathbf{P}(i, t)$ is a transition probability matrix and \mathbf{E} is a transition intensity matrix. In addition to the equation (4) we have the initial conditions:

$$\text{at } t = 0 \quad p_1(t)=1; \quad p_2(t)=p_3(t)=p_4(t)=0 \quad (5)$$

The elements of $\mathbf{P}(i, t)$ should meet the followings normalization condition

$$\sum_{i=1}^n P_1(t) = 1 \quad (6)$$

for all i (row sum). This condition is due to the fact that Markov chain events are incompatible and form a complete group.

From known elements of transition matrix \mathbf{P} and vector of initial probabilities \mathbf{p}_0 (formula (3)) are determined *absolute probabilities* of system states in time function after a fixed number of transition steps.

Application

Markovian random process models as described in 2. are universal. Random process described by the model is invariant as regards the type of structure, its material, operational environment. Degradation properties of structure elements are described by two parameters: *degradation criterion* and parameter *failures intensity* (mean rate of events occurrence)

The criterion of degradation may be adopted any factor of the stress-strain state: reliability, internal effort, deformation. In our case, the criterion of degradation assumes *the reliability of the element*, as the most common factor of the stress-strain state.

As regards failure intensity parameter, it requires special investigation in phenomenological model of failures accumulation, and its determination in our model is described below.

Problem formulation

We set a task to build a phenomenological probabilistic model of object element degradation in the process of operation. Element degradation model is aimed to establish reliability law in time function and, thus, to install tool for its technical condition prediction [9, 10]. Our developed model has two components: phenomenological classification tables of discrete states and reliability function. The following four assumptions form theoretical basis of the models proposed are shown below.

A. As criterion an element state condition a numerical reliability parameter for integrated quantitative estimation of condition assessment and prediction is assumed. The reliability of an element is defined in the classical meaning [13] as the probability that the safety limit state will be violated:

$$p_f = \text{Prob} [r(R) - s(S) \leq 0], \quad (7)$$

where $r(R)$ and $s(S)$ are functions of random variables associated with resistance and loading effects respectively

B. Lifecycle of the structures is divided into 5 discrete states. Each state is characterized by a set of quantitative and informal qualitative degradation indices which describe hierarchy of element failures.

C. Element degradation process within its lifecycle is described by a discrete Markovian random process with continuous time.

D. Transition from one discrete state to another is described as Poisson's process with discrete states and continuous time.

Discrete states of element

We will formulate the Markovian process for models in which wandering in discrete states is carried out in only one direction: from a state smaller, to a state with a larger number. Transition is possible not only to adjacent state but with overshooting to subsequent states as well. In terms of discrete Markovian process the problem comes down to search of unconditional probabilities of system S being at a random step k in state S_i . We shall treat system of failures due to object element worn-out as a flow of random discrete Markov chain events. «Random system discrete state» acts as random value.

Let us introduce 5 discrete states within element operation lifecycle which form a cortege $S = \{S_1, S_2, \dots, S_5\}$. Discrete states are described by a set of quantitative and qualitative informal indices which characterize hierarchy of element failures [7, 11]. A generalized description of states representing accumulation of failures as hierarchy of successive element failures is specified in Table 1.

Table 1

General characteristics of states

State	Characteristic of state
S_1	Element meets all project requirements
S_2	Element does not meet all project requirements, but the requirements of both first and second groups of boundary states are not violated
S_3	Element does not meet all project requirements, but the requirements of the first group of boundary states are not violated. Requirements of the second groups of boundary states may be partially violated, if this is not limiting normal structure functioning
S_4	Element has signs of violation of requirements of the first group of boundary states, whereas requirements of the second group of boundary states are seriously violated
S_5	Element has serious violations of requirements of the first group of boundary states, impossibility of their prevention is proven and its operation must be stopped

In terms of discrete Markovian process the problem comes down to search of *unconditional probabilities* of system **S** being at a random step k in state S_i , that is, to obtaining transient probability matrix.

Let us assume that an element is to be consistently in state S_1, S_2, \dots, S_n , and the transitions from one discrete condition state to another are carried out at the moment of time t_1, t_2, \dots, t_{n-1} . The global aim of this model is to establish the change reliability law in the function of time and the time prediction of stay in each subsequent discrete state. This is a discrete stochastic process with continuous time and regular distributed time intervals between states. The graph of the process represents a linear sequence of events, i.e. a transition from any state to next (fig. 1).

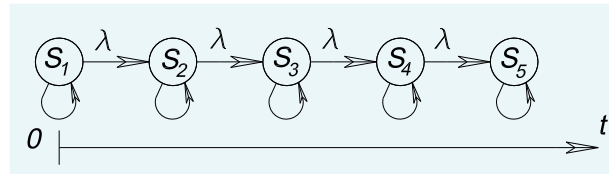


Figure 1 – Graph of the deterioration process

We determine matrix $\mathbf{P}(i, t)$ and matrix **E** by Chapman-Kolmogorov equations (equation (4)). We slightly amend matrix elements form to simplify recording. The state flow intensities independent from step and time

$$\lambda_{ij}(t) = \lambda(t) = \lambda. \quad (8)$$

Equation (5) in this case is simplified and written as:

$$\frac{d\mathbf{P}}{dt} = \lambda \mathbf{P}(i, t). \quad (9)$$

By integrating system of equations (11) we obtain transition probability matrix **P**:

$$\mathbf{P}(i, t) = \begin{bmatrix} 0,982 & 0,018 & 0 & 0 & 0 \\ 0 & 0,964 & 0,036 & 0 & 0 \\ 0 & 0 & 0,947 & 0,053 & 0 \\ 0 & 0 & 0 & 0,930 & 0,070 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (10)$$

We assume initial unconditional probability $p_k = 0.9998$ ($\beta = 3,8$. Here β is safety characteristic, numerical parameter connected to reliability by relation: $Pt = \Phi(-\beta)$, where Φ is standard normal distribution function), which corresponds to minimum normalized value of design reliability in State 1. From equation (3) we obtain the vector of unconditional probabilities of staying in the state j :

$$\mathbf{P}_j = [0,9998 \quad 0,9899 \quad 0,9703 \quad 0,9416 \quad 0,9047]^T. \quad (11)$$

By known vector of remaining in state $j = 1, 2, \dots, 5$ unconditional probabilities we determine degradation curves, that is, family of stochastic process implementations, each of them at given value of failure intensity parameter λ gives *predicted transition time* from state j to state $j + 1$ [7,11].

Reliability function

Reliability function describes element degradation process within its lifecycle, that is, connection is established between element reliability and operation time. It is postulated that degradation rate is

described by a single parameter λ – failure intensity index. This parameter is admitted as constant, time-independent $\lambda = \lambda(t)$.

We admit Poisson's distribution law as reliability function under hypothesis D. At $k = 5$ function has the form

$$P(t) = 1 - 0,008333 \cdot (\lambda(t)t)^5 e^{-\lambda(t)t}, \quad (12)$$

where P_t – probability of element transition to state k within time $t < T_k$, is probability function of service life in other terms.

Thus, at given failure intensity λ , equation (12) founds connection between element reliability P_t in i -th state and time t , which passed from beginning of operation to state $i=2, \dots, 5$. Element degradation curves under equation (12) see Fig. 2.

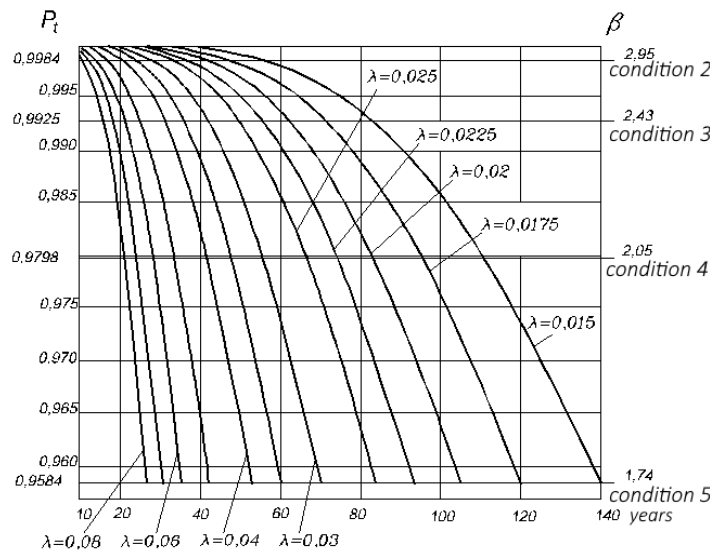


Figure 2 – Element degradation curves

Determination of flow intensity parameter

Determination of flow intensity parameter is a keystone of structure element failure accumulation under Markov phenomenological model. As seen from equation (12), a single lifecycle management parameter is the flow intensity λ . In the model under study the parameter λ is determined by solution of equation (12) with initial conditions for a particular element obtained from inspection results. The procedure of parameter λ determination was first proposed in 1999 in [152]. It consists in specific determination of initial conditions for equation (12) relative to unknown parameter λ :

- reliability $P_{t,i}$ - corresponding to determined i -th lifetime condition. This value becomes known as soon as by inspection data element discrete state is classified;
- and also t_i - time that passed from beginning of element operation to state i . Time t_i is known from bridge technical documentation.

Graphic interpretation of parameter determination procedure see Fig.3.

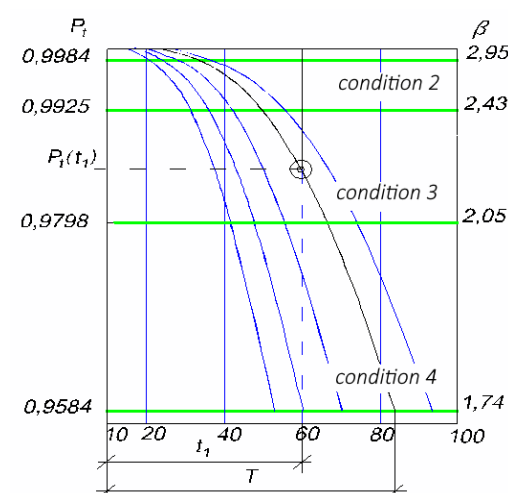


Figure 3 – Graphic interpretation of flow intensity parameter λ determination procedure

Element residual life prediction

Residual life prediction is also determined by solution of the degradation equation (12). Initial data for equation solution now become reliability $P_{i,5}$ – boundary value of reliability in operational condition 5 and flow intensity parameter λ determined at previous step from equation (12). The time T_5 obtained under such initial conditions that is time passing from element current state to fifth operational state, just is its residual life.

Algorithm of evaluation and prediction of structure element technical condition under Markovian phenomenological model of failure accumulation is specified in Table 2 – an example of operation system rules of road bridges [7].

Table 2

Algorithm of structure element lifetime condition evaluation and prediction

Step	Operation
1	<i>Conventional assessment.</i> The procedure for classifying the operating status of bridges according to the survey results of inspection and/or testing and comparison characteristic defects and damage, other signs of degradation with signs specified in the classification tables.
2	<i>Performance assessments.</i> Classification of bridge element lifetime condition from results of actual load capacity. Load capacity is determined on the basis of actual structure element dimensions, mechanical characteristics of materials and description of defects revealed in the course of inspection.
3	<i>Detail assessments.</i> Classification of bridge element lifetime condition from results of analytical calculation of their safety index actual as per moment of inspection. Initial data for safety index are inspection data with material mechanical characteristics determined for the element and quantitative degradation features of its cross-section, characteristic values of generalized resistance and load at the ultimate limit state load.
4	<i>Lifetime prediction.</i> Initial data for determination of residual life are element reliability P_i , time that passed from beginning of operation to current condition, and failure intensity flow intensity parameter λ .

Discussion

The proposed model is integral. It does not contain an explicit theoretical apparatus of an element sensitive to the material, its static scheme, construction technology, environmental conditions, and many other aspects. On the other hand, all of these factors, are taken into account in the model at a time when element state is specified by the tables of classification containing physical and mechanical signs of degradation

It is evident that the proposed technique for determining the value of the parameter λ , which manages operation lifecycle model «from present», that is, from i -th discrete state, gives full information about the loading history of the load «in the past». Flow intensity parameter λ determined in this procedure contains a lot of other information relating to object operation, such as environment characteristics, level of loading effects, features of the design and the like. As parameter λ is determined for each particular element and must be finalized after each consecutive inspection, the model becomes more accurate.

This is clear from the above formulation that the model is strictly substantiated in theory. However, as the model is phenomenological, deep penetration into physical essence of described process is necessary, as not only each of discrete states must be adequately described, but also parameter variation within one state should be correctly established, whereas the modeled process is a continuous one.

The solution on number of discrete states representing element lifecycle is quite arbitrary. It is evidently that the more discrete states, will be the more accurate description of continuous damage accumulation process. From the other side, description of a large number of discrete states requires substantial expansion of valid natural data base. Model developer shall find a reasonable compromise between these contradictory requirements.

An important theoretical aspect of model is element degradation process graph. Model graph, which depends on the number of discrete states and the relationships between them will always be the subject of special attention from the researcher, will always reflect his subjective idea of the essence and patterns of the process.

Conclusions

Markov stochastic models of damage accumulation as a result of natural deterioration, considered herein, are universal. They are well proved theoretically and have a practical orientation. These models may be applied as an effective tool for technical state assessments and prediction of structural residual resource.

A key aspect of this approach is the special procedure for evaluating the transition intensity parameter. The main lack of the model is that the basis of the elementary flow with parameter $\lambda = \text{const}$ is accepted. Actually, the parameter of failure rate is described by the stochastic function of time. However, the parameter is determined for each given element, and every time after a subsequent inspection it can be specified, so the accuracy of the model will be raised.

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МАРКОВСЬКА МОДЕЛЬ ОЦІНКИ І ПРОГНОЗУ ТЕХНІЧНОГО СТАНУ БУДІВЕЛЬНИХ КОНСТРУКЦІЙ

Анотація

Мета. Мета роботи полягає в розробці універсальної моделі оцінки технічного стану споруди в функції часу експлуатації. Теоретичним базисом розробки моделі стохастичної оцінки технічного стану споруди в процесі експлуатації, є марковська теорія випадкових процесів. Розглядаються феноменологічні моделі кумулятивного накопичення пошкодженого внаслідок природної деградації елементів в процесі життєвого циклу експлуатації. Знос елемента споруди описується марковським дискретним процесом з неперервним часом.

Деградація елементів в процесі експлуатації розглядається як потік відмов, що фізично є проявом пошкоджень елементів споруди під впливом навантажень і оточуючого середовища. Деградація елементів моста трактується як стаціонарний простійний потік пуассонського типу. Математична модель випадкового процесу з безперервним часом і дискретними станами описується відомими рівняннями Колмогорова-Чепмена. Методика. Теоретичне дослідження процесів деградації елементів мостів виконане в рамках теорії ймовірностей і математичної статистики.

Результати. Отримана модель оцінки і прогнозу технічного стану будівельних конструкцій яка базується на марковській теорії випадкових процесів. Стохастичні моделі накопичення пошкоджень внаслідок природного погіршення, розглянуті в даному документі, є універсальними.

Вони теоретично обґрунтовані і мають практичну спрямованість. Ці моделі можуть бути застосовані як ефективний інструмент для оцінки і прогнозування технічного стану та оцінки залишкового ресурсу.

Ключовим аспектом цього підходу є спеціальна процедура оцінки параметра інтенсивності відмов. Основним недоліком моделі є те, що прийнята основа елементарного потоку з параметром $\lambda = \text{const}$. В нашому випадку, параметр визначається для кожного даного елемента, і кожен раз після наступного обстеження λ отримує нове значення, тому точність моделі буде підвищуватися.

Наукова новизна. Виконане дослідження є піонерним. Вперше в системі експлуатації автодорожніх мостів пропонується стохастична оцінка технічного будівельних конструкцій.

Практичне значення. Отримана модель є практичним інструментом управління надійністю і ресурсом будівельних конструкцій.

Ключові слова: будівельні конструкції; інтенсивність відмов; марковська модель випадкового процесу, матриця переходів; процес деградації; функція надійності.